Arithmetic and Geometric Series

Sequences A sequence is a list of numbers which conform to a pattern. Each number is a *term* in the sequence. a_n refers to the nth term in the sequence.

Series When you take all the terms in a sequence and add them together, this is a *series*. s_n refers to the sum of the first n terms in a series.

Arithmetic Sequences and Series These are series where, in order to get a new term, you *add* a constant number d (the difference) to the previous term.

$$a_n = a_1 + (n-1)d$$

$$s_n = \frac{n}{2}(a_1 + a_n)$$

Geometric Sequences Series These are series where, in order to get a new term, you *multiply* the previous term by a constant number r (the ratio).

$$a_n = a_1 \cdot r^{n-1}$$

$$s_n = \frac{a_1(1-r^n)}{1-r}$$

Some geometric series *converge* to a finite number, even if you consider an infinite number of terms. In these cases, you can find the number using this equation:

$$s_\infty = \frac{a_1}{1-r}$$

Sigma Notation This is a notation used to express a sum of sequence terms. It can be used to represent many kinds of sequences, not just arithmetic and geometric sequences. It is handy when you only want to add up some subseries of a longer series.

The general form:

$$\sum_{k=1}^{n} a_k$$

This adds up all the terms between 1 and n. The variable k is the *index* variable. It serves as a placeholder in the formula for each use, for all the numbers between 1 and n. k and i are typical variable names for the index (k is for "counting" and i is for "iteration").

An example for adding up part of an arithmetic series which starts at 5 (a_1) and adds 3 each time (d):

$$\sum_{k=2}^{6} (5 + (k-1)3) = [8 + 11 + 14 + 17 + 20] = 70$$

An example for adding up part of a geometric series which starts at 3 (a_1) and multiplies by 2 each time (r):

$$\sum_{k=3}^{5} \left(3 \cdot 2^{k-1}\right) = 12 + 24 + 48 = 84$$

An example of a series which is neither arithmetic nor geometric:

$$\sum_{k=1}^{8}\left(\frac{1}{n}\right) = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8}$$

(this is a portion of the **harmonic series**)
This series will give you $\frac{\pi}{4}$. It is one of the less-efficient methods for calculating π:

$$\sum_{k=0}^{\infty}\left(\frac{(-1)^k}{2k+1}\right) = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7}... = \frac{\pi}{4}$$

Pi Notation Pi notation is just like sigma notation, only you **multiply** the terms:

$$\prod_{k=1}^{8}\left(\frac{1}{n}\right) = 1 \cdot \frac{1}{2} \cdot \frac{1}{3} \cdot \frac{1}{4} \cdot \frac{1}{5} \cdot \frac{1}{6} \cdot \frac{1}{7} \cdot \frac{1}{8}$$