Algebra II: Solving a Radical Equation

A Radical Equation $\sqrt{4x+10} - 4 = \sqrt{2x-10}$. Solve for real values of x, if any.

Solution : Our general strategy to solve radical equations is to get one radical by itself on one side of the equals sign and then square both sides. This will remove that radical. If we have more radicals, we work to get them by themselves one at a time and square both sides.

$$\sqrt{4x+10} - 4 = \sqrt{2x-10} \tag{1}$$

The $\sqrt{2x-10}$ is by itself, so we square both sides. Don't forget that you are squaring a binomial on the other side, and so this will produce more than two terms when you distribute, i.e. $(a+b)^2 = a^2 + 2ab + b^2$.

$$4x + 10 - 8\sqrt{4x+10} + 16 = 2x - 10 \tag{2}$$

Now we work to get $\sqrt{4x+10}$ by itself:

$$8\sqrt{4x+10} = 2x + 36 \tag{3}$$

This would be a good time to divide both sides by 2, although it is not neccessary:

$$4\sqrt{4x+10} = x + 18 \tag{4}$$

Now we square again:

$$16(4x+10) = x^2 + 36x + 324 \tag{5}$$

Distribute the 16 on the left:

$$64x + 160 = x^2 + 36x + 324 \tag{6}$$

Convert to a quadratic polynomial in standard form:

$$x^2 - 28x + 164 = 0 \tag{7}$$

Now we find the two roots using the quadratic formula:

$$x = \frac{28 \pm \sqrt{(-28)^2 - 4 \cdot 1 \cdot 164}}{2 \cdot 1} = 14 \pm 4\sqrt{2} = 14 - 4\sqrt{2},\ 14 + 4\sqrt{2} \tag{8}$$

We must check if one or both of these solutions are *extraneous*. One at a time we substitute each potential solution for x into the original equation everywhere x appears. If both sides of the equals sign do not agree after we simplify, then the solution is extraneous. In this case, $14 + 4\sqrt{2}$ is extraneous and $14 - 4\sqrt{2}$ is valid. Thus

$$\boxed{x = 14 - 4\sqrt{2}}$$

For the record, $14 - 4\sqrt{2}$ is approximately 8.34314575050762.