Prerequisite Knowledge - Non-Comprehensive Reference

Below you will find some handy bits of knowledge related to some of the exercises on the introductory/review quiz.

All diagrams pulled from Wikipedia.

1 Matrices - Finding the Determinant

$$\det \begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - bc,$$

Figure 1: Calculating the determinant of 2x2 matrix

$$M = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} - a_{31}a_{22}a_{13} - a_{32}a_{23}a_{11} - a_{33}a_{21}a_{12}$$

Figure 2: Calculating the determinant of 3x3 matrix.

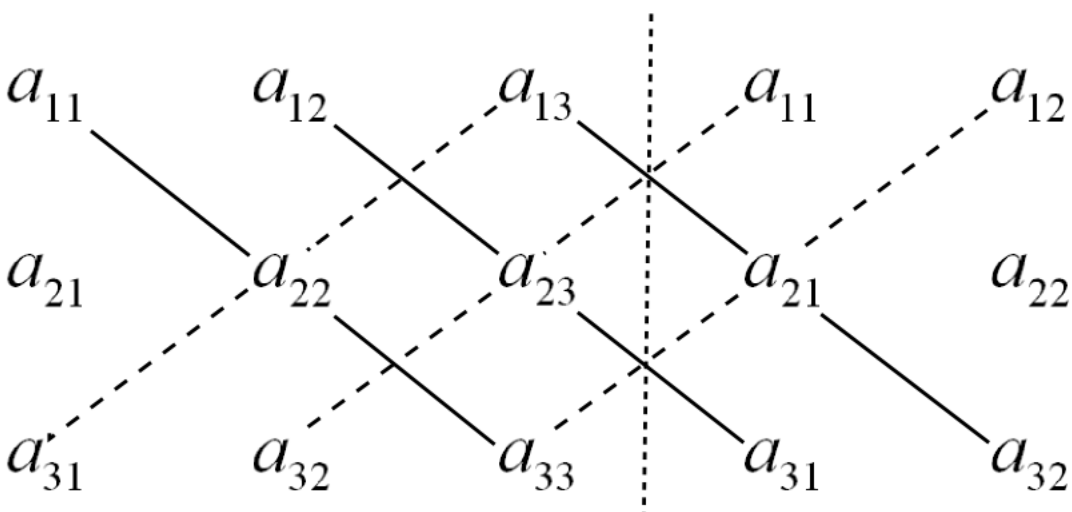

Figure 3: Remembering the 3x3 equation using the lattice method

The lattice method ("diagonal rule") works only for 3x3 matrices. You can find the determinant of **any** square matrix by using *expansion by minors*.

2 The Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The **discriminant** is $b^2 - 4ac$:

- If it is > 0, the equation has two real roots.
- If it is $= 0$, the equation has two identical roots.
- If it is < 0, the equation has complex roots of the form $Ax \pm Bi$.

3 The Triangular Definitions of Trig Functions

$$\sin\theta = \frac{opposite}{hypotenuse}$$

$$\cos\theta = \frac{adjacent}{hypotenuse}$$

$$\tan\theta = \frac{opposite}{adjacent}$$

4 30-60-90 and 45-45-90 triangles

30-60-90 triangles are a type of "right scalene" triangle. 45-45-90 triangles are "right isocelese" triangles.

The sides of a 30-60-90 triangle are related by the ratio x, $x\sqrt{3}$, $2x$.

The sides of a 45-45-90 triangle are related by the ratio x, x, $x\sqrt{2}$.

5 Circles

A standard equation for a circle which has an **origin** of (a, b) and a **radius** of r :

$$(x - a)^2 + (y - b)^2 = r^2$$

- Circumference of a circle: $2\pi r$
- Area of a circle: πr^2

6 Ellipses

A standard equation for an ellipse which has an **origin** of (a, b) and **axes** of r_1 (the vertical axis/height) and r_2 (the horizontal axis/width). Whichever axis is larger is the "major axis," and the smaller one is the "minor axis."

$$\frac{(x-a)^2}{(r_1)^2} + \frac{(y-b)^2}{(r_2)^2} = 1$$

- Area of an ellipse: $\pi r_1 r_2$

7 Spheres

- Volume of a sphere: $\frac{4}{3}\pi r^3$
- Surface area of a sphere: $4\pi r^2$

8 Triangles

Area, Method 1 $A = \frac{1}{2}bh$ (e.g., "one half base times height")

Area, Method 2 For a triangle with sides a, b, and c:

$$s = \frac{a+b+c}{2}$$
$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

9 The Slope-Intercept Form

$$y = mx + b$$

Slope Rate of change, rise over run... it's m in this equation.

Y-intercept The point where the graphed function intercepts the y-axis (thus, where $x = 0$)... it's b in this equation.

X-intercept The point where the graphed function intercepts the x-axis. If set $y = 0$ and solve for x, the value you find is the x-intercept.

10 Point-Slope Form

$$y - y_1 = m(x - x_1)$$

If you know the slope, m, of a line, and a point on the line (x_1, y_1), you can write the line's equation in this form.

11 Finding the slope

The slope is rise over run. In other words, if I have two points, (x_1, y_1) and (x_2, y_2), I can find the slope this way:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

12 The distance between two points

In essence, you use the Pythagorean theorem to find the distance between two points:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

13 Inequalities

Flip the inequality when you multiply or divide both sides by a negative:

$$-x < 4 \rightarrow x > -4$$

$$-5x < 15 \rightarrow x > -3$$

14 Functions and Inverses

- A function takes an *argument* and calculates a *value*:

  If $f(x) = 3x + 5$, and $f(2) = 11$, 2 was the argument and 11 was the value.

- Many functions can have an *inverse* function which reverses whatever the original function does. We use the notation $f^{-1}(x)$ to mean the inverse of $f(x)$.

  For example:

  $$f(x) = 3x + 5 \quad f^{-1}(x) = \tfrac{x-5}{3}$$

  So $f(2) = 11$ and $f^{-1}(11) = 2$ shows us the result of composing a function and its inverse:

  $$f^{-1}\left(f(x)\right) = x$$

15 Shifts

Consider some function f. We can tack on a number to the value (y-axis) to accomplish a vertical shift. We can tack on a number to the argument (x-axis) to accompish a horizontal shift. For a horitzontal shift, we must invert the sign.

- **Vertical Shift** $[f(x) + v]$: For example, $x^2 + 5$ shifts a parabola up 5 units.
- **Horizontal Shift** $[f(x - h)]$: For example, $(x + 2)^2$ shifts a parabola to the left two units.

16 Fractions

Division

$$\frac{3}{4} \div \frac{2}{5} = \frac{3}{4} \cdot \frac{5}{2}$$

Splitting the numerator You can split the numerator if you have terms which are being added or subtracted.

$$\frac{x+y}{z} = \frac{x}{z} + \frac{y}{z}$$

Cancelling out (Using $\frac{a}{a} = 1$)

$$\frac{x+y}{x} = \frac{x}{x} + \frac{y}{x} = 1 + \frac{y}{x}$$

$$\frac{xy}{x} = \frac{x}{x} \cdot y = 1 \cdot y = y$$

You can use this for compound terms, too:

$$\frac{(4-3x)y}{4-3x} = \frac{4-3x}{4-3x} \cdot y = y$$

A common error!:

$$\frac{xy+z}{x} \neq \frac{y+z}{x}$$

...instead the best you can do is:

$$\frac{xy+z}{x} = \frac{xy}{x} + \frac{z}{x} = y + \frac{z}{x}$$

Consolidation (Using $ax + bx = (a+b)x$)

If $3x + 4x = 7x$, then

$$3\frac{42x - 29x^2}{3y + z} + 4\frac{42x - 29x^2}{3y + z} = 7\frac{42x - 29x^2}{3y + z}$$

17 Logarithms

Remember that a logarithm is a question, and the answer to a logarithm is an exponent.

If I write $\log_b x$, I am asking "to what power do I raise b in order to get x?"

Thus, if $\log_b x = a$, then $b^a = x$.

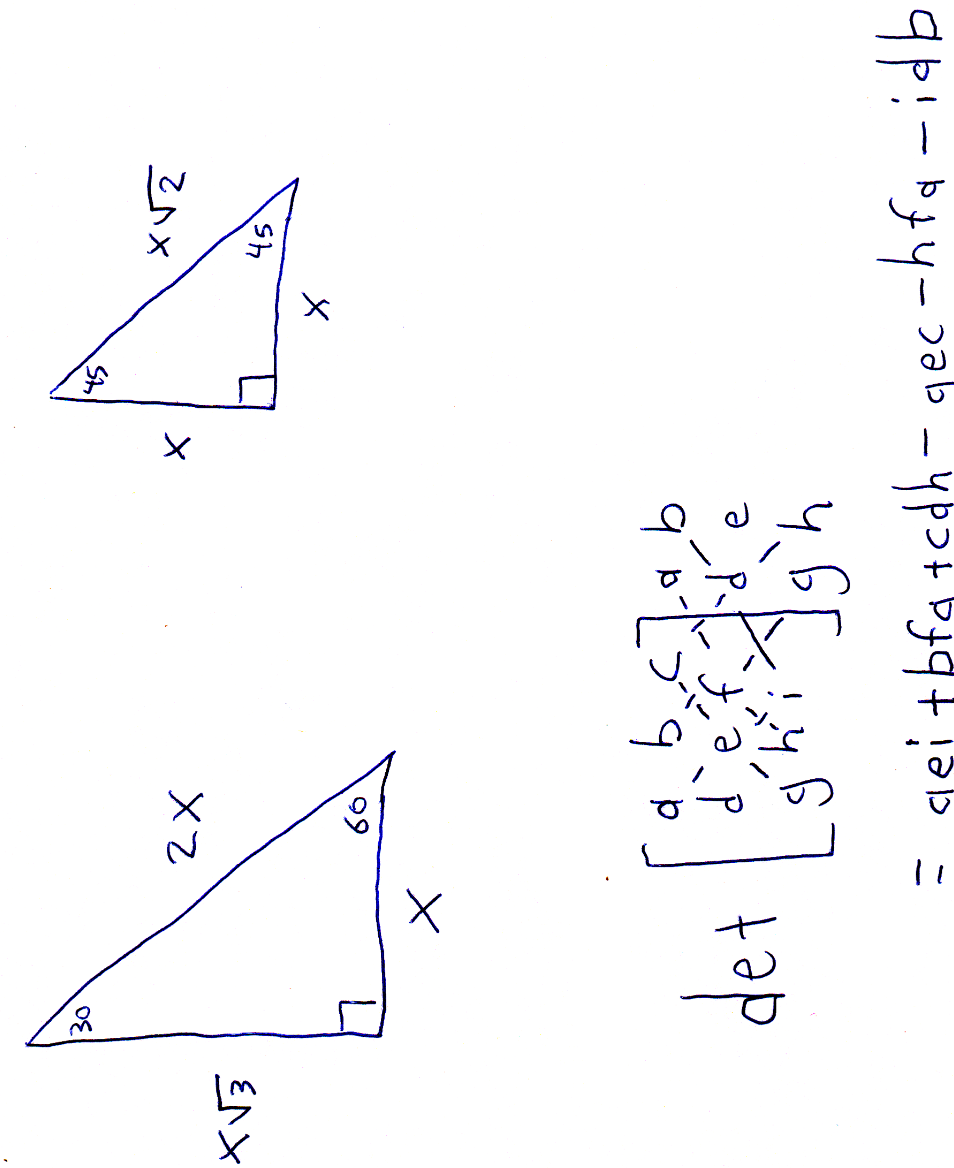

Figure 4: More diagrams